

Name: _____

MA2 Exam 1

Do and show all work on this paper. All problems must be answered using calculus techniques. Put a box around your final answers. *Scientific calculators* are permitted, but *graphing calculators* are not permitted on this exam.

1. Find $f'(x)$ given that $f(x) = 3\sin(4x)$ [10 pts.]

2. If $f(x) = 2\cot^2(3x)$, compute:

a) $f'(x)$ [6 pts.]

b) $f'\left(\frac{\pi}{12}\right)$ in simplest form [5 pts.]

c) $f''(x)$ [6 pts.]

3. Determine if the following function is continuous at $x = -2$: [9 pts.]

$$f(x) = \begin{cases} x^2 + 2x, & \text{if } x \leq -2 \\ x^3 - 6x, & \text{if } x > -2 \end{cases}$$

4. The graph at the right shows $f(x)$ on the interval $[-3, 3]$.

State all values of x on $[-3, 3]$ for which [2 pts. each]

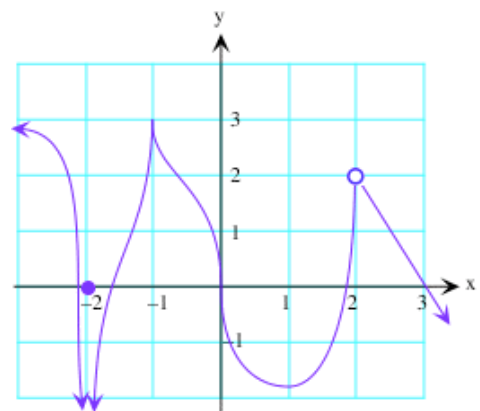
a) $f(x)$ is undefined.

b) $\lim f(x)$ does not exist.

c) $f(x)$ is discontinuous.

d) $f'(x) = 0$.

e) $f'(x)$ does not exist.



5. Given $f(x) = x^3 - 3x + 2$ [16 pts.]
- For what values of x is $f(x)$ increasing?
 - For what values of x is $f(x)$ concave up?
 - State the x -coordinate of the relative minimum.
 - State the x -coordinate of the point of inflection.
6. The gross domestic product (GDP) of a certain country is $N(t) = t^3 + 5t + 106$ billion dollars t years after 1980.
- What was the GDP in 1988? [3 pts.]
 - Find the instantaneous rate of change of the GDP, N , with respect to time, t . [6 pts.]
 - At what rate was the GDP changing in 1988? [3 pts.]
7. Find $f'(x)$ using the definition of the derivative if $f(x) = x^2 - x$. [12 pts.]
8. Find the equation of the normal line to the ellipse $2x^2 + y^2 = 57$ at the point $(-4,5)$. [14 pts.]

Extra Credit

The cardioid was developed by the Danish astronomer Ole Christensen Roemer in 1674. It was discovered during an investigation to find the best design for gear teeth and was given its name, meaning “heart-shaped,” by de Castillon in the Philosophical Transactions of the Royal Society of 1741. The following equation describes the cardioid shown below: $x^4 + 2x^2y^2 + 4x^2y + 4y^3 + y^4 - 4x^2 = 0$. Find the equation of the tangent line to the cardioid at the point $(2,0)$.

