

Exam 4 will be given on Thursday, April 27, 2006. The exam will be cumulative, with emphasis on the topics covered since the last exam.

Those topics are: trig equations (linear, quadratic), finding other trig functions given one trig function, sum and difference formulas, double-angle and half-angle formulas, identities and equations involving those formulas, inverse trig functions, and the law of cosines.

The following formulas will be given to you during the exam, so you do not need to memorize them, but you should be able to apply them to problems.

## Formulas

### Functions of the Sum of Two Angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

### Functions of the Difference of Two Angles

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

### Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

### Functions of the Half Angle

$$\sin \frac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{1}{2} A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

To prepare for the exam, in addition to reviewing your class notes and homework assignments, you should review the examples from the *June 2005 Regents Exam* (solutions are posted online) as well as the examples on this sheet.

1. If  $\sin A = -\frac{3}{5}$ ,  $\cos B = -\frac{5}{13}$ , and angles  $A$  and  $B$  are in Quadrant III, find  $\cos(A - B)$ .
2. In triangle  $ABC$ ,  $AB = 5.1$ ,  $BC = 6.4$ , and  $AC = 4.7$ . Find the measure of the largest angle to the nearest degree.
3. Find, to the *nearest degree*, all values of  $x$  in the interval  $0^\circ \leq x \leq 360^\circ$  that satisfy the equation  $3 \cos 2x + \cos x = -2$ .

4. Find all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$  that satisfy the equation  $\tan \theta + 2 \tan \theta \sin \theta = 0$ .
5. If  $\cos x = \frac{1}{8}$  where  $x$  is an acute angle, find the value of  $\cos \frac{x}{2}$ .
6. Find the exact value of  $\text{Arcsin}\left(\frac{1}{2}\right) + \text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$ .
7. A scientist was studying wave impulses. He was recording the graphs of a radio wave and an infrared wave on the same screen where  $0^\circ \leq x < 360^\circ$ . The equation  $y = 4 \cos^2 x + 1$  represents the path the radio wave travels and the equation  $y = 2 \cos x + 3$  represents the path the infrared wave travels. He wanted to record where the two impulses meet.  
Find all the values of  $x$ , in degrees, for which the two impulses meet in the interval  $0^\circ \leq x < 360^\circ$ .  
[ Only an algebraic solution will be accepted. ]
8. The amount of light that a source provides to a surface is called the *illuminance*. The illuminance  $E$  in foot candles on a surface that is  $R$  feet from a source of light with intensity  $I$  candelas is  $E = \frac{I \cos \theta}{R^2}$ , where  $\theta$  is the measure of the angle between the direction of the light and a line perpendicular to the surface being illuminated. Show that  $E = \frac{I \cot \theta}{R^2 \csc \theta}$  is an equivalent formula.
9. If  $A = 45^\circ$  and  $B = \text{Arcsin} \frac{3}{5}$ , what is the exact value of  $\csc(A - B)$ ?
10. A gardener moves around the lawn following the equation  $y = \sin x + 1$ . A fly flies around the lawn according to the equation  $y = 4 \cos(5x) + 4$ . For how many values of  $x$  on the interval  $0 \leq x \leq 2\pi$  do the gardener and the fly collide?  
(1) 5                                      (2) 8                                      (3) 9                                      (4) 10