

MA2 Final Exam Review – Sheet 2

- An object moves with velocity $v(t) = t^2 - 8t + 7$. At time $t = 0$, its position is 1.
 - Write a polynomial expression for the position of the particle at any time $t \geq 0$.
 - Write a polynomial expression for the acceleration of the particle at any time $t \geq 0$.
 - At what time(s) is the particle changing direction?

- Consider the equation $x^2 - 4xy + 4y^2 = 64$.
 - Write an expression for the slope of the curve at any point (x, y) .
 - Find the equations of the tangent lines to the curve at the point $x = 2$.
 - Find $\frac{d^2y}{dx^2}$ at $(0, 4)$.

- Water is draining at the rate of $48\pi \text{ ft}^3/\text{minute}$ from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.
 - Find an expression for the volume of water in the tank in terms of its radius at the surface of the water.
 - At what rate is the radius of the water in the tank shrinking when the radius is 16 feet?
 - How fast is the height of the water in the tank dropping at the instant that the radius is 16 feet?

- Find the area under the curve $y = 2x - x^2$ from $x = 0$ to $x = 2$ using the Midpoint Formula with $n = 2$.

- Sketch a graph of the region bounded by the curves $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$, and $x = 5$ and then find the area of the region.

6. Use a partition of $[0,2]$ with four equally-wide subintervals and right-hand endpoints to approximate the

$$\text{integral } \int_0^2 \frac{1}{x+1} dx$$

7. Let $f(x) = \begin{cases} 1+2x-x^2 & \text{if } x < 1 \\ ax+b & \text{if } 1 \leq x \leq 4 \\ \sqrt{2x+1} & \text{if } x > 4 \end{cases}$.

- (a) Find a and b so that $f(x)$ is continuous for all x .
- (b) Using your values of a and b , find $f'(1)$ or explain why it does not exist.
- (c) Using your values of a and b , find $f'(4)$ or explain why it does not exist.

8. Use the definition of the derivative for find $f'(x)$ if $f(x) = \frac{1}{x-2} + 1$.

9. Evaluate the following integrals:

(a) $\int_1^8 \frac{x^2 + x^{\frac{1}{3}}}{x} dx$

(b) $\int \frac{7x}{\sqrt[3]{2x^2-1}} dx$

(c) $\int \cos x \sqrt{\sin x} dx$

10. A closed rectangular box with a square base is to hold 96 square feet. The material for the base costs \$2 per square foot and the material for the top and sides costs \$1 per square foot. Find the dimensions of the box which will minimize the cost of materials. Justify your answer.