MA1 Homework 2, Solution to Problem 10

Find $\lim_{x\to 1} \dfrac{\sqrt{x}-x^2}{1-\sqrt{x}}$.

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1. Multiply both the numerator and denominator by the conjugate of $\sqrt{x}-x^2$.

$$\lim_{x\to 1}\left[\frac{\sqrt{x}-x^2}{1-\sqrt{x}}\cdot\frac{\sqrt{x}+x^2}{\sqrt{x}+x^2}\right]$$

2. Simplify.

$$\lim_{x\to 1}\left[\frac{x-x^4}{\left(1-\sqrt{x}\right)\left(\sqrt{x}+x^2\right)}\right]$$

3. Factor out the greatest common factor, x, in the numerator.

$$\lim_{x\to 1}\left[\frac{x\left(1-x^3\right)}{\left(1-\sqrt{x}\right)\left(\sqrt{x}+x^2\right)}\right]$$

4. Factor the difference of cubes: $1-x^3=(1-x)\left(1+x+x^2\right)$.

$$\lim_{x\to 1}\left[\frac{x(1-x)\left(1+x+x^2\right)}{\left(1-\sqrt{x}\right)\left(\sqrt{x}+x^2\right)}\right]$$

5. Multiply both the numerator and denominator by the conjugate of $1-\sqrt{x}$.

$$\lim_{x\to 1}\left[\frac{x(1-x)\left(1+x+x^2\right)}{\left(1-\sqrt{x}\right)\left(\sqrt{x}+x^2\right)}\cdot\frac{\left(1+\sqrt{x}\right)}{\left(1+\sqrt{x}\right)}\right]$$

6. Simplify.

$$\lim_{x\to 1}\left[\frac{x(1-x)\left(1+x+x^2\right)\left(1+\sqrt{x}\right)}{\left(\sqrt{x}+x^2\right)(1-x)}\right]$$

7. Reduce.

$$\lim_{x\to 1}\left[\frac{x\cancel{(1-x)}\left(1+x+x^2\right)\left(1+\sqrt{x}\right)}{\left(\sqrt{x}+x^2\right)\cancel{(1-x)}}\right]$$

$$\lim_{x\to 1}\left[\frac{x\left(1+x+x^2\right)\left(1+\sqrt{x}\right)}{\sqrt{x}+x^2}\right]$$

8. Substitute 1 for x.

$$=\frac{1\left(1+1+(1)^2\right)\left(1+\sqrt{1}\right)}{\left(\sqrt{1}+(1)^2\right)}$$

$$=\frac{1(3)(2)}{2}$$

$$=3$$