Applied Maximum/Minimum Problems – Sheet 2

1) Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

2) Build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen?

3) An open rectangular box with square base is to be made from 48 ft^2 of material. What dimensions will result in a box with the largest possible volume?

4) A container in the shape of a right circular cylinder with no top has surface area $3\pi \text{ ft}^2$. What height h and base radius r will maximize the volume of the cylinder?

5) A sheet of cardboard 3 ft by 4 ft will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume?

6) Consider all triangles formed by lines passing through the point $\left(\frac{8}{9}, 3\right)$ and both the x- and y-axes. Find the dimensions of the triangle with the shortest hypotenuse.

7) Find the point (x, y) on the graph of $y = \sqrt{x}$ nearest the point $(4, 0)$.

8) A cylindrical can is to hold $20\pi \text{ m}^3$. The material for the top and bottom costs $\$10/\text{m}^2$ and material for the side costs $\$8/\text{m}^2$. Find the radius r and height h of the most economical can.

9) You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time ?

10) Construct a window in the shape of a semi-circle over a rectangle. If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having largest possible area?

11) A right circular cylinder is to be designed to hold 22 cubic inches of a soft drink (approx. 12 fl. oz.). Find the dimensions of the cylinder that will require the least amount of material to construct.

12) A cylindrical can, open at the top, is to hold 500 cm^3 of liquid. Find the height and radius that minimize the amount of material needed to manufacture the can.