

Applications of Antidifferentiation

1. Find a function f such that $f'(x) = \sqrt{3x+1}$ and $f(1) = 5$.
2. Find a function f such that $f'(x) = 6 - 5 \sin 2x$ and $f(0) = 3$.
3. Find $f(x)$ if $f'(x) = \frac{2}{\sqrt{x}}$ and $f(1) = 6$.
4. Write y as a function of x .
 - a) $\frac{dy}{dx} = 9x^2 - 4x + 5$ if $y = 0$ when $x = -1$.
 - b) $\frac{dy}{dx} = x^2 + \sqrt{x}$ if $y = 1$ when $x = 1$.
 - c) $\frac{dy}{dx} = \cos x + \sin x$ if $y = 1$ when $x = \pi$.
 - d) $\frac{dy}{dx} = \frac{x}{\sqrt{x^2 + 1}}$ if $y = 2$ when $x = 0$.
5. Write the equation of the curve whose slope at every point is $3x^2$ and which passes through the point $(2, 6)$.
6. Write the equation of the curve whose slope at every point is $3x^2 + 2$ and which passes through the point $(1, -1)$.
7. Write the equation of the curve whose slope at every point is $3x^2 + 2x + 1$ and which passes through the point $(1, 0)$.
8. Find $f(x)$ if $f''(x) = x + 2$, $f'(0) = 3$, and $f(0) = -1$.
9. Find the equation of a curve such that $y'' = 6x - 8$ and, at the point $(1, 0)$, $y' = 4$.

10. A particle moves along the x -axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is -9 , and its position is -27 .

- Find $v(t)$, the velocity of the particle at any time $t \geq 0$.
- For what value of $t \geq 0$ is the particle moving to the right?
- Find $x(t)$, the position of the particle at any time $t \geq 0$.

11. A particle moves along the x -axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t - 18$. At any time $t \geq 0$, $v(0) = 24$ and $x(1) = 20$.

- Write an expression for the velocity, $v(t)$, of the particle at any time t .
- For what value of t is the particle at rest?
- Write an expression for the position of the particle, $x(t)$, at any time t .
- Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

12. A particle moves along the x -axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 4 \cos(2t)$. At any time $t \geq 0$, $v(0) = 1$ and $x(0) = 0$.

- Write an equation for the velocity, $v(t)$, of the particle at any time t .
- Write an equation for the position of the particle, $x(t)$, at any time t .
- For what value of t , $0 \leq t \leq \pi$, is the particle at rest?

13. A particle, initially at rest, moves along the x -axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

- For what value(s) of t is the particle at rest?
- Write an equation for the position of the particle, $x(t)$, at any time $t \geq 0$.
- Find the total distance traveled by the particle from $t = 0$ to $t = 2$.