The Intermediate Value Theorem

Given the function $f(x) = x^3 - 3x^2 + 2x - 1$, complete the table of values below:

x	-1	0	1	2	3	4
$f(x)$						

Between what two x-values can we guarantee that $f(x)$ has a root? Why?

1. Suppose that f is a continuous function defined for all real numbers x and $f(-5) = 3$ and $f(-1) = -2$.

   If $f(x) = 0$ for one and only one value of x, then which of the following could be x?

   (A) –7 (B) –2 (C) 0 (D) 1 (E) 2

2. If $f(x) = x^3 - x + 3$ and if c is the only real number such that $f(c) = 0$, then c is between

   (A) –2 and –1 (B) –1 and 0 (C) 0 and 1 (D) 1 and 2 (E) 2 and 3

3. State whether each statement given below is

   • False

   • True, but not due to the Intermediate Value Theorem

   • True due to the Intermediate Value Theorem

   (a) If $g(x)$ is a function with $g(0) = 4$ and $g(3) = 7$, then there is some x between 0 and 3 such that $g(x) = 5$.

   (b) Let $f(x) = x^3 + 2x^2 - 5$. There is some value of x between 1 and 2 where $f(x) = 4$.

   (c) If $f(t)$ is continuous on the closed interval $[-5, 3]$, $f(-5) = 6$, and $f(3) = -9$, then $f(0)$ could be equal to 10.

4. (a) The function $f(x) = \sin x$ never takes on the value 2 on the closed interval $[0,\ 2\pi]$. Why does this not contradict the Intermediate Value Theorem?

   (b) The function $f(x) = \dfrac{x+1}{x-1}$ is zero at $x = -1$ and has the value 3 at $x = 2$. Yet there is no x for which $f(x) = 1$. Why does this not contradict the Intermediate Value Theorem?